# Beneficial use of spectral broadening resulting from the nonlinearity of the fiber-optic channel

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We discuss the possibility of exploiting spectral broadening resulting from fiber nonlinearity for the transmission of information. The spectral broadening induced by nonlinearity combined with the appropriate waveform can turn QAM-like constellations into frequency-shift-keying (FSK) constellations over a much larger dimension. Thus the Kerr effect can be thought of as a large dimensional mapper/modulator. A simple single-span fiber-optic link implemented over dispersion shifted fiber is assumed for the demonstration of the principle. It is shown that for a particular constellation the achievable data-rates in the presence of nonlinearity can be significantly higher than the capacity characterizing a linear channel with the same input bandwidth.

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Optical nonlinearities play a central role in fiber-optic telecommunications systems. While in most cases, the effects of fiber-nonlinearity are adverse, they can be beneficially exploited in some situations, as was shown for example in [1], [2], and as we show here in a different context. In this paper we propose a scheme where Kerr nonlinearity is used in order to produce broad-band optical modulation while deploying a narrow-band modulator. Specifically, a standard external optical modulator is followed by a nonlinear fiber that expands the bandwidth of the transmitted signal. Signal detection at the end of the link can then be performed by means of an array of narrow-band receivers, each covering a specific band of frequencies. The achievable communications rates are significantly higher than those that could be achieved if signals produced by the same modulator were to be injected directly into a linear optical channel, although they are realized over a notably broader bandwidth.

As the goal of this Letter is to illuminate an interesting principle in the theory of communications, we choose the simplest possible configuration and ignore the many important details involved in a practical optical communications link. Clearly, in a realistic multi-span link scenario, it may be very difficult to predict and rely upon the often varying transmission properties involving nonlinearity, dispersion and polarization effects. We will assume a link in which nonlinearity is known and predictable and where (in addition to nonlinearity) the received signal is impaired only by additive Gaussian noise. This situation can only be considered as marginally realistic in links consisting of a single span with negligible residual dispersion so as to avoid the effects of intersymbol interference. Alternatively one could think of the nonlinearity as produced by a highly nonlinear element which is introduced into the transmitter immediately following the modulator and prior to injection into the link. In the latter configuration one must assume that the nonlinearity of the link is negligible and that all linear impairments are compensated for at the receiver. Finally, in order to avoid unnecessary complications that may obscure the principle that we wish to convey, we will assume the use of a dispersion shifted fiber for which a simple analytical relation between the input and output fields exists.

Under all of the above assumptions the relation between the signal waveform $x(t)$ produced by the modulator and the waveform $y(t)$ which arrives at the receiver is given by [3]

$$y(t) = x(t)e^{j\gamma L|x(t)|^2} + n(t) \tag{1}$$

where $\gamma$ is a constant representing the effect of nonlinearity and $L$ is the fiber length. We find it convenient to introduce the parameter $\kappa = \gamma L E[|x|^2]$ in order to characterize the overall strength of nonlinearity, where $E[|x|^2]$ denotes the averaged input signal power. The term $n(t)$ accounts for the presence of additive Gaussian noise.

In what follows we evaluate the proposed scheme in terms of the data-rate that can be extracted from it. The pulse-shape that we chose to work with is the square-root of a triangular pulse,

$$g(t) = \frac{2\sqrt{E_g}}{T_s}\begin{cases} \sqrt{t} & t \in [0, T_s/2] \\ \sqrt{T/2 - t} & t \in [T_s/2, T_s] \\ 0 & t < 0 \text{ or } t > T_s \end{cases},$$

where $T_s$ is the symbol duration, and $E_g$ is its energy. The rational behind choosing this waveform is that upon squaring $x(t)$ (as implied by Eq. (1)) it will produce an efficient bilateral frequency shift of the signal, as we discuss in what follows. The constellation points that we use to modulate this waveform are

$$c_{n,k} = \sqrt{n} \cdot \exp(kj\pi/4),$$

where $n$ scales the energy of the transmitted symbol and $k$ sets the value of the optical phase. We transmit with integer $n$ values between 1 and $N$ and with 4 phases corresponding to $k \in [0, 1, 2, 3]$, thereby producing a QAM-like constellation for $x(t)$, as illustrated in Fig. 1.

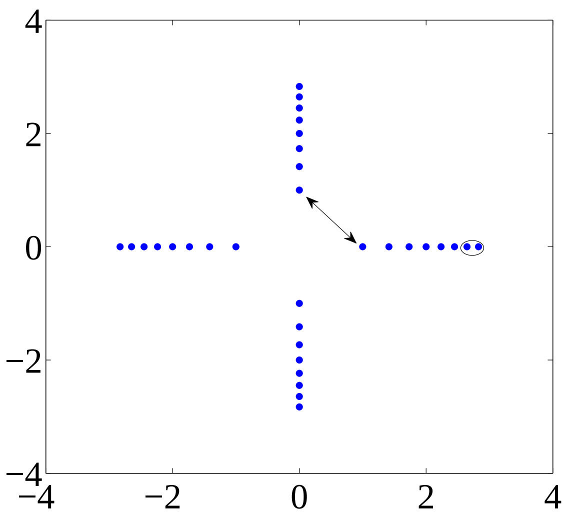

Fig. 1. The values of $C_{n,k}$ used in the signal constellation with $n$ in the range of 1 to 8. Minimal distance pair for $\kappa = 0$ is circled. The arrow marks the minimal distance pair for large $\kappa$

Upon propagation through the nonlinear fiber, the rising and falling edges of the waveform $g(t)$ are frequency shifted in opposite directions, such that constellation points corresponding to different values of $n$ are shifted to nearly orthogonal frequency bands. This phenomenon is illustrated in Fig. 2 in the time domain and in and in the frequency domain in Fig. 3, where the different curves represent the output spectra for different values of $n$. While the input signal resides in a two-dimensional signal space, the output signal (for large enough $\kappa$) can be viewed as residing in a $2N$-dimensional signal space, since every value of $n$ is mapped to a different two dimensional signal space.

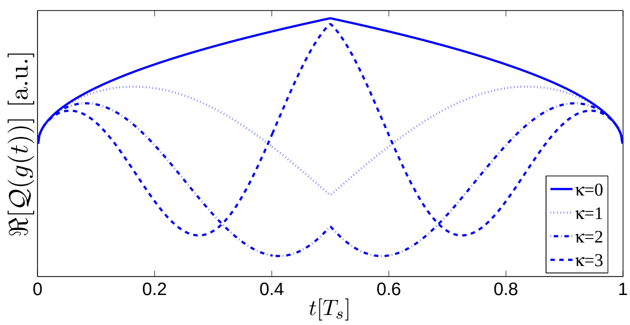


Fig. 2. The real part of the electric field of the signal after passing through increasing levels of nonlinearity.

As we assume that the detected signal is affected by additive Gaussian noise, the relevant figure of merit is the minimum Euclidean distance between the received waveforms, which can be expressed as

$$d_{min}^2 = \min \int_0^T \Big| \mathcal{Q}[c_{n,k} g(t)(t)] - \mathcal{Q}[c_{m,l} g(t)] \Big|^2 dt, \quad (2)$$

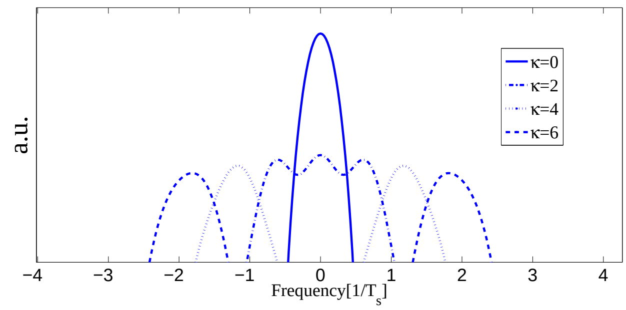


Fig. 3. The power spectrum of the signal after passing through increasing levels of nonlinearity.

where $\mathcal{Q}(x) = x \exp(i\gamma L|x|^2)$ is the nonlinear operation included in Eq. (1), and where the minimization in (2) is performed over all values of $(n, k)$ and $(m, l)$ that are not equal to each other. Maximization of the minimum distance in an additive Gaussian noise channel is the simplest, albeit not rigorously the best, approach for maximizing the data-rate.

In Fig. 4 we show the minimum distance normalized to the average symbol energy $\mathcal{E}_s$ as a function of the nonlinearity parameter $\kappa$.

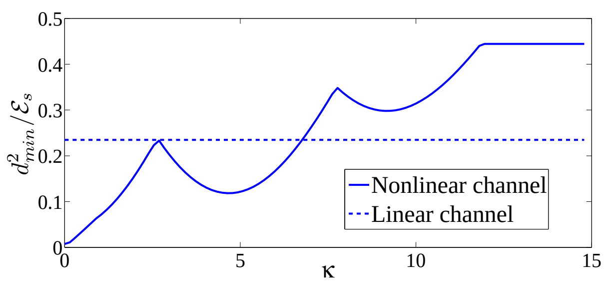


Fig. 4. Minimal distance normalized to the average symbol energy $\mathcal{E}_s$ for the 32-point constellation shown in Fig. 1 ($k = 1, 2, 3, 4$ and $n$ in the range of 1 to 8). The dashed horizontal line is an upper bound for the minimum distance that can be achieved with 32 points in the absence of spectral broadening [4]. The graph saturates at $0.44 = 2E_g n_{min}/\mathcal{E}_s = 2/4.5 = 0.44$.

The calculation was performed with the constellation shown in Fig. 1, where $k = 1, 2, 3, 4$ and $n$ is between 1 and 8, producing a total of 32 different values for $c_{nk}$. The average symbol energy of this constellation is $\mathcal{E}_s = 4.5E_g$. Note that prior to applying the nonlinearity the minimum distance corresponds to constellation points that have the same value of $k$, but adjacent $n$ values. It is proportional to $\sqrt{n+1} - \sqrt{n}$ and

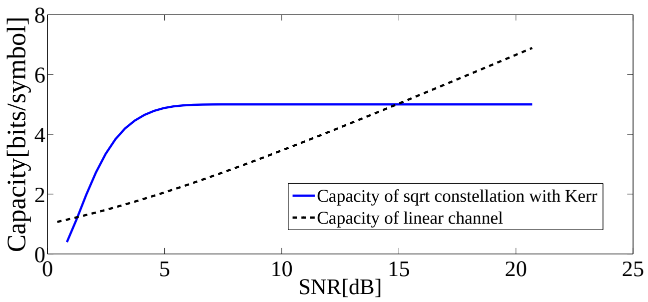


Fig. 5. Solid curve is the achievable data rate with the constellation shown in Fig. 1. The dashed curve shows the Shannon capacity in the original bandwidth, i.e. without nonlinear broadening.

decays for large values of $n$ as $2n^{-1/2}$. Conversely, in the limit of very strong nonlinearity, when symbols having different values of $n$ are mapped into nearly orthogonal frequency bands, the minimal distance is equal to $\sqrt{2E_g n_{\min}}$, where $n_{\min}$ is the smallest $n$ value in the constellation. This simply corresponds to the distance between the points $(n_{\min}, k)$ and $(n_{\min}, k+1)$ in the original constellation (marked by an arrow in Fig. 1). The asymptotic square-minimal distance is therefore twice the energy of the least energetic symbol. In the limit where the minimum value of $n$ in the constellation points that are used for transmission is very large (implying peak to average power ratio approaching unity), $d^2_{min}$ will approach twice the average symbol energy. For reference we note that it can be shown that in the absence of nonlinearity, the square-minimum distance with 32 constellation points cannot be smaller than $d^2_{min} = 0.235\mathcal{E}_s$ [4], a value that is illustrated by the flat horizontal line in the Fig. 4. Note that while the initial distance (for $\kappa = 0$) in our constellation is much lower than this value, it exceeds it considerably as soon as $\kappa$ becomes larger than 7. Moreover, the minimal distance to the neareast neighbor for constellation points that do not belong to the inner most ring (most of the points are not part of the inner ring) is considerably larger. Hence, the gain is bigger than the ratio between the graphs of Fig. 4. This of course occurs at the expense of a correspondingly considerable spectral broadening.

In Fig. 5 we show the achievable rate of our scheme for $\kappa \approx 8$ together with Shannon's capacity formula applied to the initial bandwidth of our signal (i.e. the bandwidth of $x(t)$). To obtain the achievable rate we evaluate all the distances between constellation points and apply the Blahut-Arimoto algorithm [5] for evaluating the maximum mutual information. Consistently with Fig. 4, the throughput of the nonlinear system rapidly exceeds Shannon's capacity saturating at the level of $\log_2(32) = 5$ bits.

To conclude, we note that the scheme discussed in this Letter is an interesting manifestation of Shannon's famous argument stating that nonlinear mapping from one dimension to many can help create a more robust communications system [6]. Unlike the spirit of Shannon's original work the nonlinear mapping in the case of the optical fiber channel can be attributed to the transmission medium itself, a situation that is not frequently encountered in communications. The presence of spectral broadening and in particular the case of extreme spectral broadening which we consider here, may also reflect on the studies of the capacity of the fiber-optic channel, most of which [7], [8] concentrated on a regime where nonlinear broadening of the signal is negligible. We suspect that construction of schemes that allow operation in the opposite regime may produce interesting and relevant new insights. For a different way to evaluate mutual information in the optical-fiber channel and other nonlinear integrable channels, see [9, 10].

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